Notes 10.5 & 10.6 Determinants & Inverses

I = Identity Matrices:

2^{nd} order → $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3^{rd} order

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4^{th} order **etc…**

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Each square matrix has a determinant, which is a single numerical value. If the determinant is 0, then the inverse of the matrix does not exist (dne)

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is written as } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

These straight line symbols indicate determinant

The product of a matrix (A) and its inverse (A^{-1}) is the identity matrix:

$$A \bullet A^{-1} = I \quad \text{or} \quad A^{-1} \bullet A = I$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

matrix

$$A^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Determinant →

- **3 x 3 determinant (3rd order)**

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

"expansion by minors"

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

↑ always subtract the middle term

WARM-UP

A. Write the identity matrix for a 2x2 and a 3x3 matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

WARM-UP

B. Evaluate:

$$\begin{vmatrix} 6 & -3 \\ 2 & 3 \end{vmatrix} = 6(3) - 2(-3)$$

$$= 18 + 6$$

$$= \boxed{24}$$

Same type of question

there is an inverse

C. Find the determinant of the matrix

$$\begin{bmatrix} -9 & 3 \\ 2 & -\frac{2}{3} \end{bmatrix} = -\frac{9}{1}\left(-\frac{2}{3}\right) - 2(3)$$

$$= 6 - 6$$

$$= \boxed{0}$$

inverse does not exist (dne)

WARM-UP

D. Find the inverse of matrix M

$$M = \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$$

$$M^{-1} = \frac{1}{6(3)-2(-3)} \begin{bmatrix} 3 & 3 \\ -2 & 6 \end{bmatrix}$$

$$= \frac{1}{18+6} \begin{bmatrix} 3 & 3 \\ -2 & 6 \end{bmatrix}$$

$$= \frac{1}{24} \begin{bmatrix} 3 & 3 \\ -2 & 6 \end{bmatrix}$$

or

$$= \begin{bmatrix} \frac{1}{8} & \frac{1}{8} \\ -\frac{1}{12} & \frac{1}{4} \end{bmatrix}$$

WARM-UP

E. Find the determinant of matrix N.

$$\det(N) = \begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix} = 2\begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} - 3\begin{vmatrix} 0 & 4 \\ -2 & 6 \end{vmatrix} + -1\begin{vmatrix} 0 & 2 \\ -2 & 5 \end{vmatrix}$$

$$= 2(12-20) - 3(0+8) + -1(0-4)$$

$$= 2(-8) - 3(8) + -1(4)$$

$$= -16 - 24 - 4 = \boxed{-44} \text{ det}$$